On a solution of the multidimensional truncated moment problem on the vertices of the hypercube

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Abstract

Let d be an arbitrary positive integer, $\mathbb{N}_0^d$ be the set of all d-tuples of non-negative integers, and for any $\alpha = (\alpha_1, \ldots, \alpha_d) \in \mathbb{N}_0^d$ let $|\alpha| = \alpha_1 + \cdots + \alpha_d$. If n is a non-negative integer and K a closed subset of $\mathbb{R}^d$, the truncated K-moment problem examines the existence of a non-negative measure μ, supported on K, such that, for a given (real valued) sequence $\{s_\alpha\}_{0 \le |\alpha| \le n}$,

$$s_\alpha = \int_K x^\alpha \, \mathrm{d}\mu(x)\,, \text{ for any } \alpha \in \mathbb{N}_0^d \text{ such that } |\alpha| \le n.$$

The talk deals with the case of K being the set of the vertices of the d-hypercube, i.e. $K_d = \{\pm 1\}^d$. The group structure of K_d and a simple linear optimisation argument prove to be powerful enough so that necessary and sufficient conditions for a solution are obtained when $n \ge d$, as well as sufficient conditions when $n < d$. Finally, a simple separation argument from functional analysis, combined with the previous results, yields a concrete description of the set of non-negative polynomials on K_d of degree at most n, for any $n \ge d$. The talk is based on a joint work with C. Emary and D. Kimsey.