The dynamic generalization of the Eshelby problem: the waves of phase change

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Abstract

The physics that govern waves emanated from expanding regions of phase change are manifested in deep focus earthquakes, where under high pressure the material undergoes phase weakening with the resulting loss of strain energy being partially converted into kinetic. A fundamental analytical wave solution of physics is presented for a self-similarly (subsonically) expanding ellipsoidal region in which the material undergoes phase change as it expands. The waves emitted are the pressure (P) and shear (S) ones, and those (M) due to the ellipsoidal surface of discontinuity. The interior domain of the expanding inclusion is a lacuna, where the particle velocity vanishes and the interior stress is constant. Thus, these waves constitute the dynamic generalization of the seminal Eshelby ellipsoidal inclusion problem (isotropic and anisotropic), with the static Eshelby inclusion obtained as a particular limit. In another limiting case, where the expanding ellipsoid becomes flat, and the transformation strain large as to give a rate of displacement discontinuity, and as a consequence of the constant stress interior property, the dynamic generalization of the Eshelby problem yields the solution of the expanding elliptical crack of Burridge and Willis that can emit Rayleigh waves with zero energy-release rate. The "driving force" required for this motion to occur under applied loading is presented.

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